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The MPC-in-the-Head Framework and the Limbo Protocol

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0 Outline

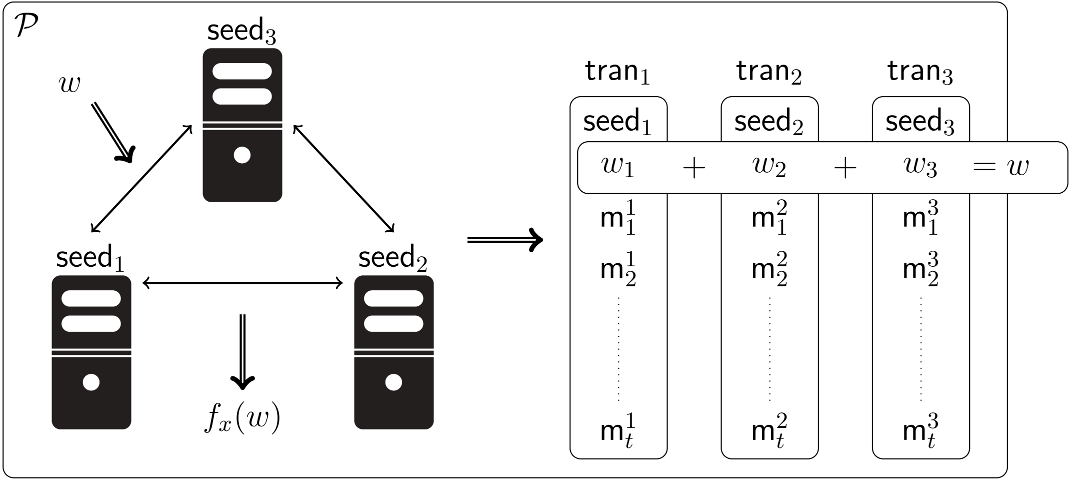
1 Outline

1 ZKPoK from MPC in the head

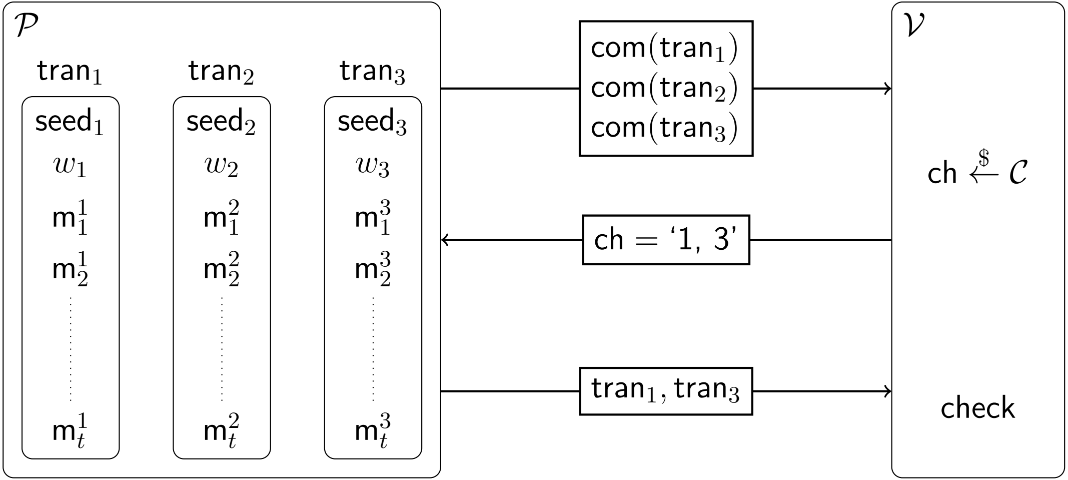
- Want efficient ZKPoK for arbitrary NP relation $\mathcal{R}$.
- Given
  1. x: public statement
  2. w: $\mathcal{P}$'s private witness,

  want to convince $\mathcal{V}$ that $\mathcal{R}(x, w) = 1$ without revealing w.
- [IKOS07]: multiparty computation (MPC) of $f_x(w) = \mathcal{R}(x, w)$.
  Simulated by $\mathcal{P}$ in the head and checked by $\mathcal{V}$.

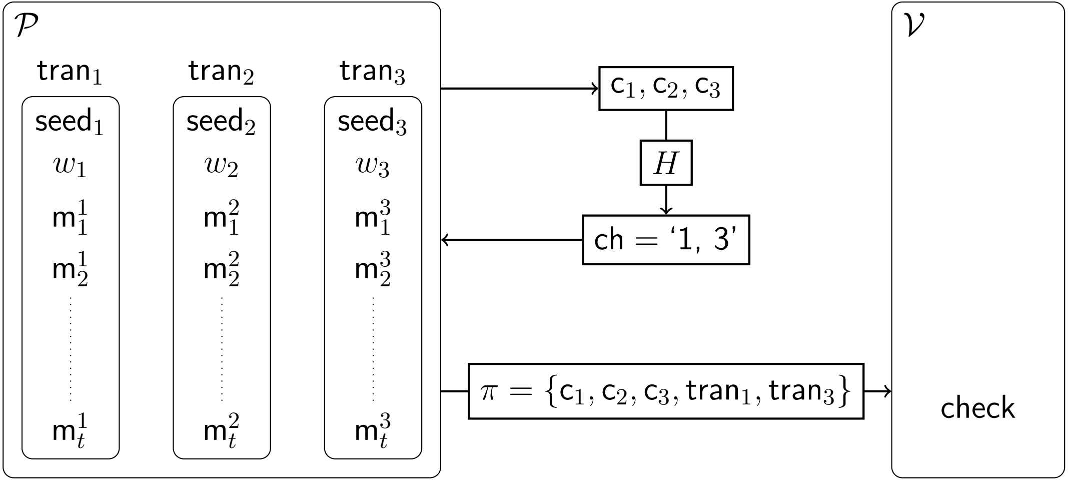
1 MPC in the head?



1 ZKPoK from MPC in the head



1 NIZKPoK from MPC in the head



1 Properties of the MPCitH Proof System

- *Correctness:* If MPC protocol for $f_x(w)$ is correct, then so is MPCitH.

- *Soundness:* If $f_x(w) \neq 1$ and opened parties show $f_x(w) = 1$ then $\mathcal{P}$ cheated during MPC protocol.
  - Assuming com is *binding*, $\mathcal{P}$ can't cheat for opened parties.
  - Assuming com is *binding*, $\mathcal{P}$ must cheat on hidden party *before* it sees ch.

  Soundness error is exactly $\frac{1}{N}$. In practice $N = 2, 4, 8, 16, 32, 64$.

- *Zero-knowledge:* First, com must be *hiding*.
  Second, $\mathcal{V}$ sees $N - 1$ transcripts, so MPC protocol must be $(N-1)$-*private*.

1 Optimizations in Practice

- Commitments $\mathsf{com}_1, \ldots, \mathsf{com}_N$ can be compressed with a Merkle tree.
  This means sending only 1 hash value in the first round, instead of N.

- Because $\frac{1}{N}$ is not cryptographically secure, the proof can be repeated in parallel with different seeds to increase soundness exponentially.

- Opening $N - 1$ transcripts can use a lot of data (and make a large proof). Using MPC protocol in the *broadcast model* means

  $$\text{"revealing } \mathsf{tran}_1, \mathsf{tran}_3\text{"} \approx \text{"sending } \mathsf{server}_2\text{'s output;"}$$

  everything else can be computed by $\mathcal{V}$ from the seeds.

1 MPCitH from MPC with Preprocessing

- Some MPC protocols use preprocessing / online paradigm.
  - Preprocessing generates *input-independent correlated randomness*.
  - Online phase uses it for *low communication* and *input-dependent* computation.
- This can be used for MPC in the head.
  - **aux**: *correlated randomness* without preprocessing communication included in π.
    So more correlated randomness = bigger proof π.

1. $\mathcal{P}$ simulates and commits to many preprocessing executions.
2. $\mathcal{V}$ challenges all-but-some of them to open and verify.
   Indep. of w, so $\mathcal{P}$ reveals both master seed and correlated rand **aux**.
3. With the rest, $\mathcal{P}$ simulates and commits to online executions.
4. $\mathcal{V}$ challenges and verifies as before; uses less data as online phase is cheap.

2 Outline

2 The Picnic Signature Scheme [CDG+17, ZCD+20]

Given block cipher $F_k(\boldsymbol{x}) : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$,

- Gen: $\boldsymbol{x} \xleftarrow{\$} \mathcal{X} = \{0,1\}^{\kappa}$, $k \xleftarrow{\$} \mathcal{K} = \{0,1\}^{\kappa}$, $\boldsymbol{y} \leftarrow F_k(\boldsymbol{x})$.

$$\mathsf{sk} = k \text{ and } \mathsf{pk} = (\boldsymbol{y}, \boldsymbol{x}).$$

  Security: function $G_{\boldsymbol{x}} : k \mapsto F_k(\boldsymbol{x})$ is OWF with respect to k.

- Sign: Given message m, compute $\sigma = \pi \leftarrow \mathsf{Prove}_m(f_{\boldsymbol{y},\boldsymbol{x}}(k))$.

$$f_{\boldsymbol{y},\boldsymbol{x}}(k) = \{F_k(\boldsymbol{x}) \stackrel{?}{=} \boldsymbol{y}\}$$

- Verify: verify the proof, including m in the challenge computation.

Choice of F_k: LowMC [ARS+15] as a binary circuit;

2 Picnic: MPC for binary circuits I [CDG+17, ZCD+20]

$f_x(w)$ as binary circuit C with wires and gates (XOR and AND).

- Wire α, real value is z_α, masked as $\hat{z}_\alpha = z_\alpha \oplus \lambda_\alpha$ for random $\lambda_\alpha \in \{0, 1\}$.
- $[\lambda_\alpha]$ is n-out-of-n XOR secret sharing

$$\lambda_\alpha = \bigoplus_{i=1}^{n} \lambda_\alpha^{(i)} \in \{0, 1\}.$$

- Each party P_i holds $\hat{z}_\alpha$ and a share $\lambda_\alpha^{(i)}$.
- Preprocessing prepares masks, online phase computes masked values $\hat{z}_\alpha$.

2 Picnic: MPC for binary circuits II

Preprocessing:

- For each *input* or *AND gate output* wire α, random mask $[\lambda_\alpha]$ from seed.
- For XOR gate $\gamma = \alpha \oplus \beta$, set $[\lambda_\gamma] = [\lambda_\alpha] \oplus [\lambda_\beta]$; local $=$ free.
- For AND gate $\gamma = \alpha \cdot \beta$, need $[\lambda_{\alpha,\beta}]$, where $\lambda_{\alpha,\beta} = \lambda_\alpha \cdot \lambda_\beta$; not free.
- P_i can set $\lambda^{(i)}_{\alpha,\beta}$ at random, for $i \in \{1, \ldots, N-1\}$, but P_N needs

$$\lambda^{(N)}_{\alpha,\beta} = \lambda_{\alpha,\beta} - \bigoplus_{i=1}^{N-1} \lambda^{(i)}_{\alpha,\beta}.$$

- This $\lambda^{(N)}_{\alpha,\beta}$ has to be computed by $\mathcal{P}$ and added to aux;
  1 bit/AND gate added to π.

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2 Picnic: MPC for binary circuits III

Online:

- Public reconstruction of $[\lambda_\alpha]$ is done by broadcast of each $\lambda_\alpha^{(i)}$;
- Parties begin with masked $\hat{z}_\alpha$ given by $\mathcal{P}$ and $[\lambda_\alpha]$ from seed.
- Computation proceeds by computing $\hat{z}_\gamma$ for each gate $(\alpha, \beta) \to \gamma$ in C.
- XOR gate: $\hat{z}_\gamma = \hat{z}_\alpha \oplus \hat{z}_\beta$; local = free.
- AND gate: locally compute

$$[s] = \hat{z}_\alpha[\lambda_\beta] \oplus \hat{z}_\beta[\lambda_\alpha] \oplus [\lambda_{\alpha,\beta}] \oplus [\lambda_\gamma],$$

reconstruct s (1 bit of communication per party),
and compute $\hat{z}_\gamma = s \oplus \hat{z}_\alpha \hat{z}_\beta$.

2 The AES Algorithm

AES is a 128-bit state block-cipher with key length of 128, 192 or 256 bits. The round function is composed of four operations on the state:

1. AddRoundKey
2. SubBytes – only non-linear component, aka S-box
3. ShiftRows
4. Mix Columns

SubBytes applies the function $s \mapsto \begin{cases} s^{-1} & \text{if } s \neq 0 \\ 0 & \text{o/w} \end{cases}$ over $\mathbb{F}_{2^8}$, followed by a public affine transformation.

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2 BBQ: MPC for arithmetic circuit I

$f_x(w)$ represented as arithmetic circuit C with values and gates ($+$ and $\times$). [dDOS19, BN20]

- Wire value $x \in \mathbb{F}$ is randomly shared as $\langle x \rangle = (x^{(1)}, \ldots, x^{(n)})$ such that

$$x = \sum_{i=1}^{n} x^{(i)}.$$

  $\langle x \rangle$ is n-out-of-n additive sharing of x.
- Each party P_i holds only a share $x^{(i)}$.
- Preprocessing prepares shared randomness.
  Online phase computes $+$ and $\times$ gates.

2 MPC for arithmetic circuit II

Operations with public constants are free, and so are additions between shared values (because of additive sharing).

Only multiplication $\langle z \rangle = \langle x \cdot y \rangle$ costs preprocessing and communication.

Given *triple* $(\langle a \rangle, \langle b \rangle, \langle c \rangle)$ such that $c = a \cdot b$, multiplication is:

1. Compute $\langle \alpha \rangle = \langle x - a \rangle$ and $\langle \beta \rangle = \langle y - b \rangle$.
2. Open α and β.
3. Locally compute $\langle z \rangle = \langle c \rangle - \alpha \cdot \langle b \rangle - \beta \cdot \langle a \rangle + \alpha \cdot \beta$.

Computing $\langle c \rangle$ adds $\log_2 |\mathbb{F}|$ bits to aux in π as P_N needs

$$c^{(N)} = a \cdot b - \sum_{i=1}^{N-1} c^{(i)}.$$

2 Computing inversion in $\mathbb{F}$ for AES

Given S-box input $\langle s \rangle$ and random $\langle r \rangle$:

1. Compute $\langle s \cdot r \rangle$.
2. Open $s \cdot r$.
3. Locally compute $\langle s^{-1} \rangle = (s \cdot r)^{-1} \cdot \langle r \rangle$.

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3 Outline

1. NIZKPoK from MPC in the Head with Preprocessing
2. MPCitH from Circuit Computation: Picnic and BBQ
3. MPCitH from Circuit Verification: Banquet and Limbo
   - Witness Extension and Verification
   - Banquet: Verification of Multiplications (and Inverses)
   - Limbo: Improved Multiplication Verification

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3 Witness extension and verification

Idea from sacrificing techniques in MPC

- Prover "injects" the results of multiplications—no need to compute.
  - The witness is *extended* with the outputs of non-linear gates.
- MPC parties execute a *verification* protocol—batching possibilities.
  - e.g. Sacrifice one "suspicious" operation to verify another.

ZKPoK protocol sketch [BN20]

MPC parties receive "suspicious" multiplication results and verify them by sacrificing "suspicious" random triples ⇒ no cut & choose.

$$0 \stackrel{?}{=} \langle v \rangle = \epsilon \langle z \rangle - \langle c \rangle + \alpha \langle b \rangle + \beta \langle a \rangle - \alpha \cdot \beta$$

3 Increased Number of Rounds

- In order to *soundly* sacrifice triples, the parties need a *random challenge* ϵ.
- This challenge comes from $\mathcal{V}$, *after* suspicious data is committed.
- *After*, $\mathcal{P}$ still has to commit to MPC execution of sacrificing *verification*.
  After receving ϵ, the MPC parties continue to check $v \stackrel{?}{=} 0$.

This yields a protocol with 5 or more rounds:

1. $\mathcal{P}$ commits to (views of) suspicious data;
2. $\mathcal{V}$ sends sacrificing (i.e. verification) challenge;
3. $\mathcal{P}$ commits to (views of) data verification protocol;
4. $\mathcal{V}$ sends party-opening challenge (as usual);
5. $\mathcal{P}$ opens selected parties.

3 Banquet: Verification of Inverses

$\mathcal{P}$ injects m "suspicious" inverses $t = s^{-1}$, so MPCitH parties have pairs (s, t) such that $s \cdot t = 1$ *allegedly*.

Naïve verification protocol

For the ℓ-th inverse operation:

1: Set multiplication tuple $(s_\ell, t_\ell, 1)$.
2: Sacrifice with triple (a, b, c).

This is expensive: each multiplication requires 1 correlated triple, and 1 sacrifice. $4|C| + 1$ elts. in total.

3 Banquet: Polynomial-based Verification I

Define polynomials S, T and $P = S \cdot T$ as:

$$\begin{array}{ccc} S(1) = s_1 & T(1) = t_1 & P(1) = s_1 \cdot t_1 = 1 \\ \vdots & \vdots & \vdots \\ S(m) = s_m & T(m) = t_m & P(m) = s_m \cdot t_m = 1 \end{array}$$

Check $P \stackrel{?}{=} S \cdot T$:

1: Sample random $R \leftarrow \mathbb{F} \setminus \{1, \ldots, m\}$;
2: Open $P(R), S(R), T(R)$
3: Check

$$P(R) \stackrel{?}{=} S(R) \cdot T(R).$$

3 Banquet: Polynomial-based Verification II

Lemma (Schwartz–Zippel)

Let $Q \in \mathbb{F}[x]$ be non-zero of degree $d \geq 0$; for any $\mathbb{S} \subseteq \mathbb{F}$, $\Pr_{R \leftarrow \mathbb{S}}[Q(R) = 0] \leq \frac{d}{|\mathbb{S}|}$.

- Here, $Q = P - S \cdot T$; non-zero iff $t_\ell \neq s_\ell^{-1}$ for some ℓ.
- Opening $S(R)$, $T(R)$ leaks information $\Rightarrow$ add random points $S(0), T(0)$.
- P (and also Q) is of degree $d = 2m$ and $|\mathbb{S}| = |\mathbb{F} - m|$, so

$$\Pr_{R \leftarrow \mathbb{S}}[Q(R) = 0] \leq \frac{2m}{|\mathbb{F} - m|}.$$

3 Polynomial-based Verification III

Improved protocol

1. Prover commits to S (randomized) and T.
2. Prover commits to P.
3. MPC parties open $Q(R) = P(R) - S(R) \cdot T(R)$, for random R.

$2|C| + 4$ elts.; no cut & choose, no triple. Actually one triple, but hidden!

(Extra randomness $r_1, \ldots, r_m$ in S prevents further cheating.)

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3 Generalized polynomial-based checking I

Previous protocol verifies:

$$\begin{pmatrix} r_1 s_1 & \cdots & r_m s_m \end{pmatrix} \begin{pmatrix} t_1 \\ \vdots \\ t_m \end{pmatrix} \stackrel{?}{=} \sum_{\ell=1}^{m} r_\ell.$$

Now, let $m = m_1 \cdot m_2$, and instead verify:

$$\begin{pmatrix} r_1 s_{1,k} & \cdots & r_{m_1} s_{m_1,k} \end{pmatrix} \begin{pmatrix} t_{1,k} \\ \vdots \\ t_{m_1,k} \end{pmatrix} \stackrel{?}{=} \sum_{j=1}^{m_1} r_j, \qquad k \in \{0, \ldots, m_2 - 1\}.$$

($s_{j,k}$ and $t_{j,k}$ are rearranged from s_ℓ and t_ℓ.)

3 Generalized polynomial-based checking II

Define S_j and T_j as

$$\begin{aligned} S_j(k) &= r_j \cdot s_{j,k} & T_j(k) &= t_{j,k} & k \in \{0, \ldots, m_2 - 1\} \\ S_j(m_2) &= \bar{s}_j & T_j(m_2) &= \bar{t}_j; \end{aligned}$$

and let $P = \sum_{j=1}^{m_1} S_j \cdot T_j$.
Generalized verification protocol

1. Prover commits to S_j (randomized) and T_j;
2. Prover commits to P;
3. MPC parties open $Q(R) = P(R) - \sum_{j=1}^{m_1} S_j(R) \cdot T_j(R)$, for random R;

Total: $= |C| + O(\sqrt{|C|})$, instead of $2|C|$.

3 Inner-product compression

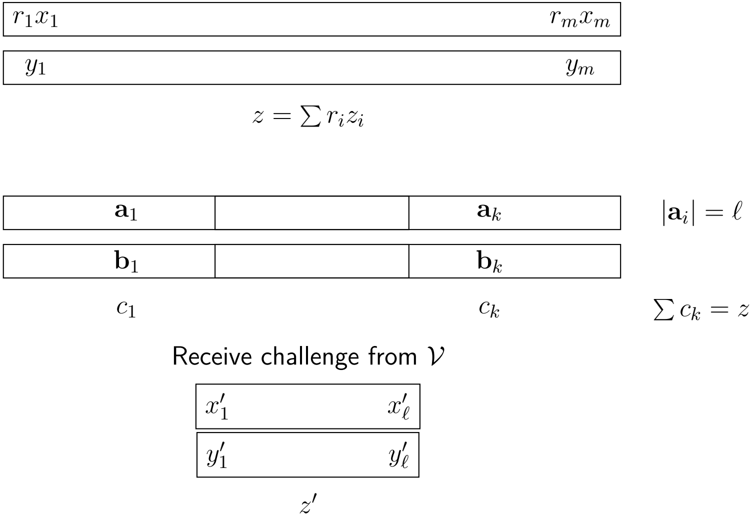
Limbo [dOT21]: several compression rounds.

Checking

$$\langle \tilde{\mathbf{x}}, \mathbf{y} \rangle \stackrel{?}{=} z$$

becomes

$$\langle \mathbf{x}', \mathbf{y}' \rangle \stackrel{?}{=} z'.$$



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